Experiment 5: The Ultimate Speed

<u>Object:</u> To test the validity of the Newtonian and relativistic kinetic energy formulas for objects moving near the speed of light.

<u>Apparatus</u>: The apparatus is a particle accelerator. Since we would have to save up the annual physics budget for about five thousand years to purchase one, we will watch a movie of the experiment being performed at MIT instead. (A real movie, not a video. Very retro.)

Electrons emitted by a hot wire are accelerated by an electric field from a Van de Graff generator. Measuring the accelerating voltage tells us the kinetic energy this field gives the electrons. In two trials, their energy is double checked by measuring the heat they put into the target they eventually hit. The electrons are timed as they move a measured distance down a long evacuated pipe, so their speed can be determined. From the measured energy, you will calculate the speed predicted by each formula, then compare to the speed that was actually measured.

As explained in Phy 132, voltage is related to energy: If an electron moves between two points which differ by one volt, then the electron gains 1 electron-volt of energy. The electron-volt is a unit of energy equal to 1.60×10^{-19} joule.

<u>Procedure:</u> For each of the five runs of the experiment, record the energy of the electrons and their measured speed.

<u>Calculations</u>: (It saves you converting the kinetic energy into joules if you use the fact that the electron's rest energy is $mc^2 = .511$ MeV.) Using the kinetic energy measured in each of the five trials, compute the electrons' speed in both of the following ways:

1. From the Newtonian formula,

$$KE = \frac{1}{2}mv^{2} \Rightarrow \qquad v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2KEc^{2}}{mc^{2}}} \Rightarrow \qquad v_{N} = c\sqrt{\frac{2KE}{mc^{2}}}$$

2. From the relativistic formula,

$$KE = (\gamma - 1)mc^{2} \implies \underline{KE} = \gamma - 1 \implies \underline{KE} + 1 = \underline{1} = \frac{1}{\sqrt{1 - (v/c)^{2}}} \implies \left(\frac{KE}{mc^{2}} + 1\right)^{2} = \frac{1}{1 - (v/c)^{2}} \implies 1 - \left(\frac{v}{c}\right)^{2} = \frac{1}{(KE/mc^{2} + 1)^{2}} \implies 1 - \frac{1}{(KE/mc^{2} + 1)^{2}} = \left(\frac{v}{c}\right)^{2} \implies v_{R} = c\sqrt{1 - \frac{1}{\left(\frac{KE}{mc^{2}} + 1\right)^{2}}}$$

Plot a graph of speed as a function of kinetic energy showing all three data sets: experimental, Newtonian formula, and relativistic formula. When plotting different sets of points on the same graph, you can clarify which are which by plotting one set as \bigcirc , another as \bigcirc , and the third as \triangle , with a key telling which is which. The point (0, 0) should be included on all three curves, since when a speed of zero means a kinetic energy of zero by definition.

In your conclusion, compare each set of computed speeds with those measured in the experiment. For each formula, in what situations does it work?

KE (MeV)	Measured v (m/s)	Calculated $v_n (m/s)$	Calculated $v_r (m/s)$

Sample calculation of v_n:

Sample calculation of v_r:

(Attach graph.)
